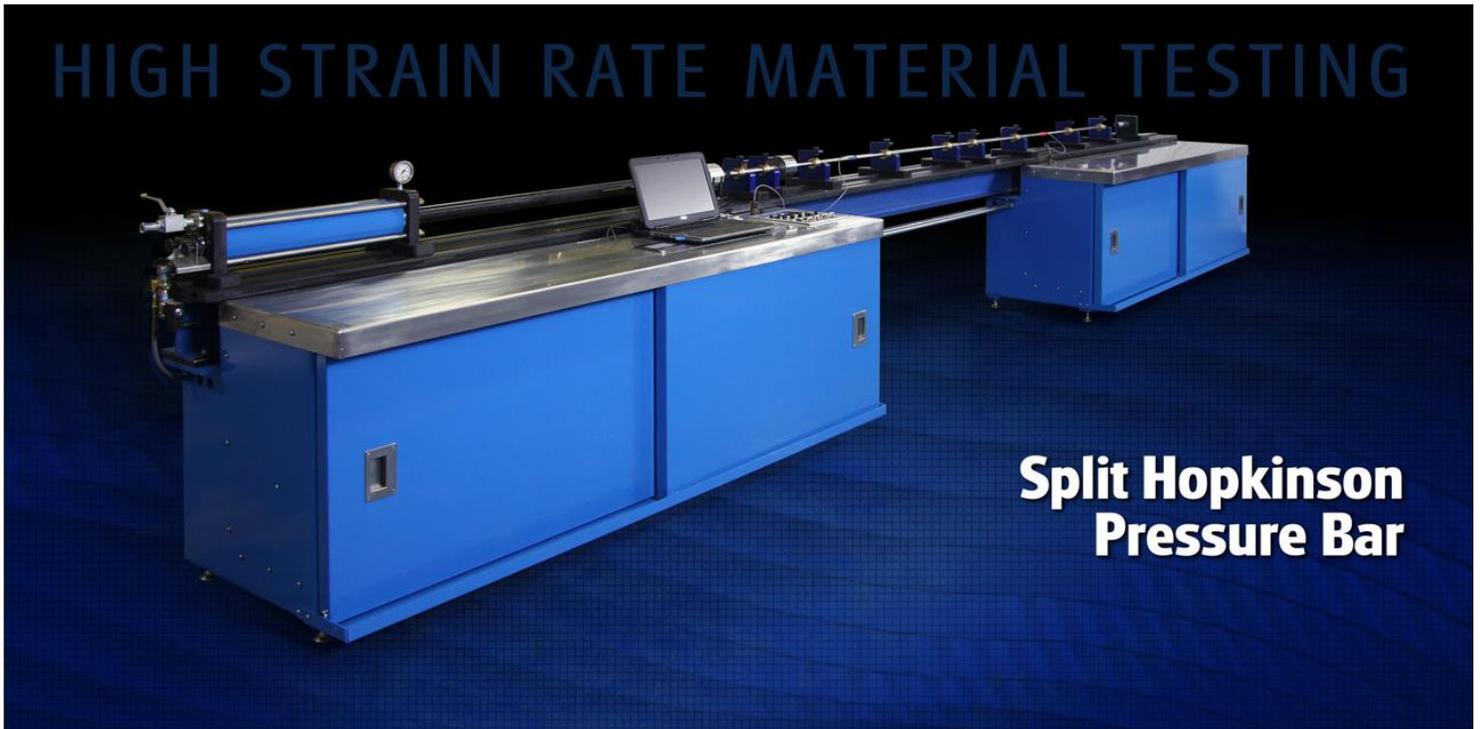HIGH STRAIN RATE MATERIAL TESTING

Split Hopkinson Pressure Bar

REL SURE-pulse

SHPB Equations used in SURE-Pulse

F   SURE-Pulse® Calculations: Descriptions & Examples

SURE-_pulse_ Calculations

SURE-pulse uses several calculations in reaching its results.

Young's modulus of bar

The Young's modulus of the bar is found from the wave speed and the density.

The wave speed is found in the bar calibration by using the distance between the strain gauges and the time between pulses.

$$Wave\ speed = \frac{Distance\ between\ strain\ gauges}{Time\ Between\ Pulses}$$

For example, if the pulse travels a distance of 1 m in 201 μs in a steel bar, the wave speed can be found as follows.

$$Wave\ speed = \frac{1\ m}{201.2\ \mu s} = 4970\ \frac{m}{s}$$

The Young's modulus is found from the wave speed and density.

$$Youngs\ Modulus = Wave\ Speed^2 * Density$$

For example, if the same steel bar that was used above has a density of 8100 $\frac{kg}{m^3}$, the Young's modulus can be found as follows.

$$E = \left(4970\ \frac{m}{s}\right)^2 * 8100\ \frac{kg}{m^3} = 200\ GPa$$

Strain calculation in bar

The strain gauge readings in volts need to be converted into strain. This requires the resistance of the strain gauge, the resistance of a calibrating resistor, the gauge factor of the strain gauge, and the voltage to which system is calibrated. (Vishay Micro-Measurements, 2002) Please read the Vishay manual for further details on strain calculation.

$$Strain = Voltage\ reading * \frac{Gauge\ Resistance}{Voltage\ Calibrated * Gauge\ Factor * (Gauge\ Resistance + Calibration\ Resitance)}$$

For example if the voltage reading on the strain gauge is 2V, the gauge resistance is 120 Ω, the voltage the gauge was calibrated at was 2V, the gauge factor is 2, and the calibrating resistor has a resistance of 59940 Ω the strain in the bar can be found as follows.

$$Strain = 2\ V * \frac{120\ \Omega}{2\ V * 2 * (120\ \Omega + 59940\ \Omega)} = .001$$

Sample Stress and Strain Calculations

Force on the sample is equal to the transmitted force in the pulse transmission bar if the sample is in equilibrium. To find the stress in the sample, the ratio of areas of the bar and the sample is multiplied by the stress in the bar.

$$\sigma_{sample}(t) = \frac{Cross\ Sectional\ Area\ Of\ Bar}{Cross\ Sectional\ Area\ Of\ Sample} E_\tau \varepsilon_\tau(t)$$

Where E_τ is the elastic modulus of the transmission bar, and $\varepsilon_\tau(t)$ is the strain in the transmission bar at time t.

For example, with the a 2 cm steel bar and a cylindrical sample with a diameter of .5 cm, for a strain of .001 in the transmission bar, the corresponding stress in the sample can be found as follows.

$$\sigma_{sample}(t) = \frac{.00314\ m^2}{.0000196\ m^2} * 200\ GPa * .001 = 800\ MPa$$

The strain rate of the sample $\dot{\varepsilon}_{sample}(t)$ can be found using one dimensional wave theory and an assumption of equilibrium.

$$\dot{\varepsilon}_{sample}(t) = -2\ \frac{Wave\ Speed\ Incident\ Bar}{Length\ of\ Sample} \varepsilon_r(t)$$

Where $\varepsilon_r(t)$ is the strain of the reflected pulse in the incident bar at time (t).

For example, with the above mentioned steel bar and a sample of 1 cm, for a strain of .001 in the reflected pulse in the incident bar, the corresponding strain in the sample can be found as follows.

$$\dot{\varepsilon}_1(t) = -2\ \frac{4970\frac{m}{s}}{.01\ m}.001 = 994\frac{1}{s}$$

Sample strain can be easily found from strain rate using an integral.

$$\varepsilon_{sample}(t) = \int_{t_{begin}}^{t_{end}} \dot{\varepsilon}_{sample}(t)dt$$

Time is indexed from the beginning of the pulses, so the first strain gauge reading that is determined to be in each selected pulse is at $t = 0$ and is considered to occur at the same time in the sample.

If you assume that the above mentioned strain rate is constant over 6 μs, the sample strain can be found as follows.

$$\varepsilon_1(t) = \int_0^{6\ \mu s} 994\frac{1}{s}dt\ = .05964$$

Dynamic Force

The force on the back face can be found from the transmitted strain $\varepsilon_\tau(t)$

$$Back\ Face\ Force = Area\ Of\ Bar * E_\tau \varepsilon_\tau(t) \quad e.g.\ .0003142\ m^2 * 200\ GPa * .001 = 62840\ N$$

The force on the front face can be found from the difference in the strains of the incident $\varepsilon_I(t)$ and reflected $\varepsilon_R(t)$ pulses.

$$Front\ Face\ Force = Area\ Of\ Bar * E_I\big(\varepsilon_I(t) - \varepsilon_R(t)\big)$$

For example, with a 2 cm steel bar with a Young's modulus of 200 GPa, a strain of .001 on the incident pulse, and a strain of .0009 on the reflected pulse the force on the front face of the sample can be found as follows.

$$.0003142\ m^2 * 200\ GPa * (.001 - .0009) = 6284\ N$$

Noise Issues

Often the incident bar has a good deal of background noise due to the incident pulse having travelled through the bar. In order in minimize bias in the sample from noise, the voltage of the reflected pulse is adjusted down by the initial voltage.

$$Voltage\ adjusted = Voltage\ Measured - Voltage\ baseline$$

If noise is low (2.5% of signal or less), then the $Voltage\ baseline \approx 0$ and the result is no adjustment.

Energy Calculations

Bar alignment and shot quality can be evaluated using energy calculations. If the bars are closely aligned, the energy of the shot should stay level as it travels down the bar. Also, the energy from a non-trapped shot should closely match the energy from the striker bar.

There are two forms of energy in the bar, elastic and kinetic, and both are proportional to ε_B^2. More details on these numbers can be found in (Chen & Song, 2011) in the Conventional Bar section. The power travelling through the bar at any point in time can be found with the following equation.

$$Power(t) = \left(\frac{1}{2}\rho_B A_B C_B^3 + \frac{1}{2}A_B C_B E_B\right)\varepsilon_B^2(t)$$

For example, if you have a 2 cm steel bar that at some point in the pulse experiences a strain of .001, the power at that point can found as follows.

$$\left(\frac{1}{2} * 8100\ \frac{kg}{m^3} * .0003142\ m^2 * \left(4970\frac{m}{s}\right)^3 + \frac{1}{2} * .0003142\ m^2 * 4970\frac{m}{s} * 200\ GPa\right) * (.001)^2$$
$$= 312375\ watts$$

Where ρ_B is the density of the bar, A_B is the area of the bar, C_B is the wave speed of the bar, E_B is the Young's modulus of the bar, and ε_B is the strain in the bar.

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To find the energy of a pulse, Integrate power over the whole pulse

$$Pulse\ Energy = \int_0^{end\ of\ pulse} Power(t)dt$$

For example, a pulse in the steel bar with a strain of .001 over 60 μs can be found as follows using the power calculated above.

$$Pulse\ Energy = \int_0^{60\ \mu s} 312375\ watts\ dt = 18.74\ joules$$

Number of Reflections

The above equations assume that the sample is in equilibrium, it has been shown (Ravichandran & Subhash, 1994) that 4 reflections of the wave in the sample are necessary to reach equilibrium. The number of reflections in a sample are calculated as follows.

$$Number\ of\ Reflections = \frac{1}{2}\frac{Duration\ of\ Pulse}{l_s/C_s}$$

$$e.g.\ Number\ of\ Reflections = \frac{1}{2}\frac{60\ \mu s}{.01\ m/6420\frac{m}{s}(Aluminum)} = 19.3$$

Where the duration of the pulse is the duration of the transmitted pulse, l_s is the length of the sample, and C_s is the wave speed of the sample.

References:

Chen, W., & Song, B. (2011). *Split Hopkinson (Kolsky) Bar*. Springer Science.

Ravichandran, G., & Subhash, G. (1994). Critical Appraisal of Limiting Strain Rates for Compression Testing of Ceramics in a Split Hopkinson Bar. *Journal of American Ceramics*, 263-267.

Vishay Micro-Measurements. (2002). *2300 System Signal Conditoning Amplifier 2311 Instruction Manual.* Raleigh: Vishay.